Waves & Acoustics

Lecture-III

For

4th Semester (General)

Paper-DSC1DT

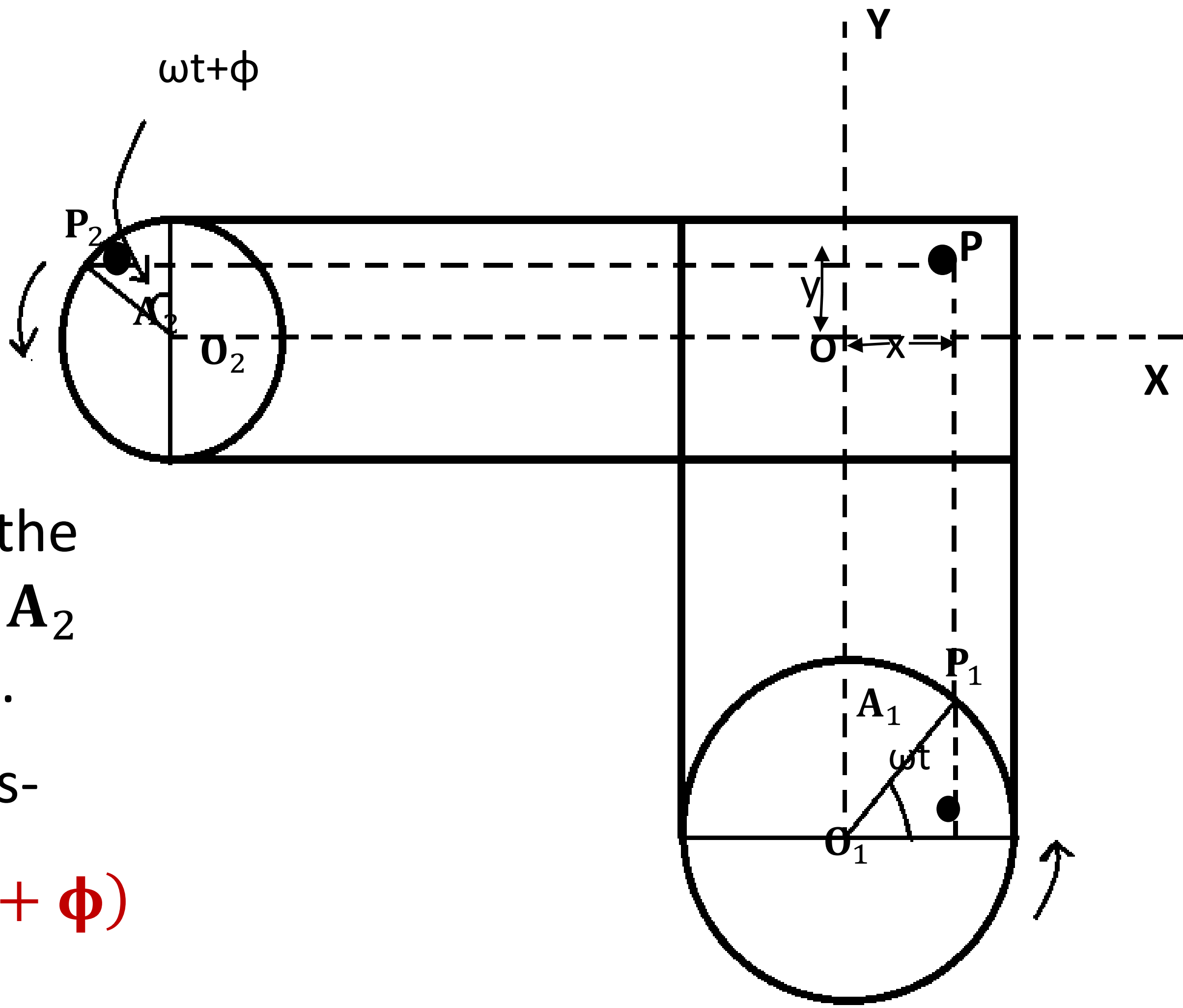
Department of Physics

SMHGGDCW

Superposition of Two Perpendicular Harmonic Oscillations With equal frequencies

GRAPHICAL METHOD:

From the fig. we can see that the two perpendicular SHM is represented as two circle by using the rotating vector method as we have seen earlier.



The circle of radius $\mathbf{A_1}$ centred at $\mathbf{O_1}$ represents the oscillation along x-axis where The circle of radius $\mathbf{A_2}$ centred at $\mathbf{O_2}$ represents the oscillation along y-axis.

From the geometry it easy to write the equations as-

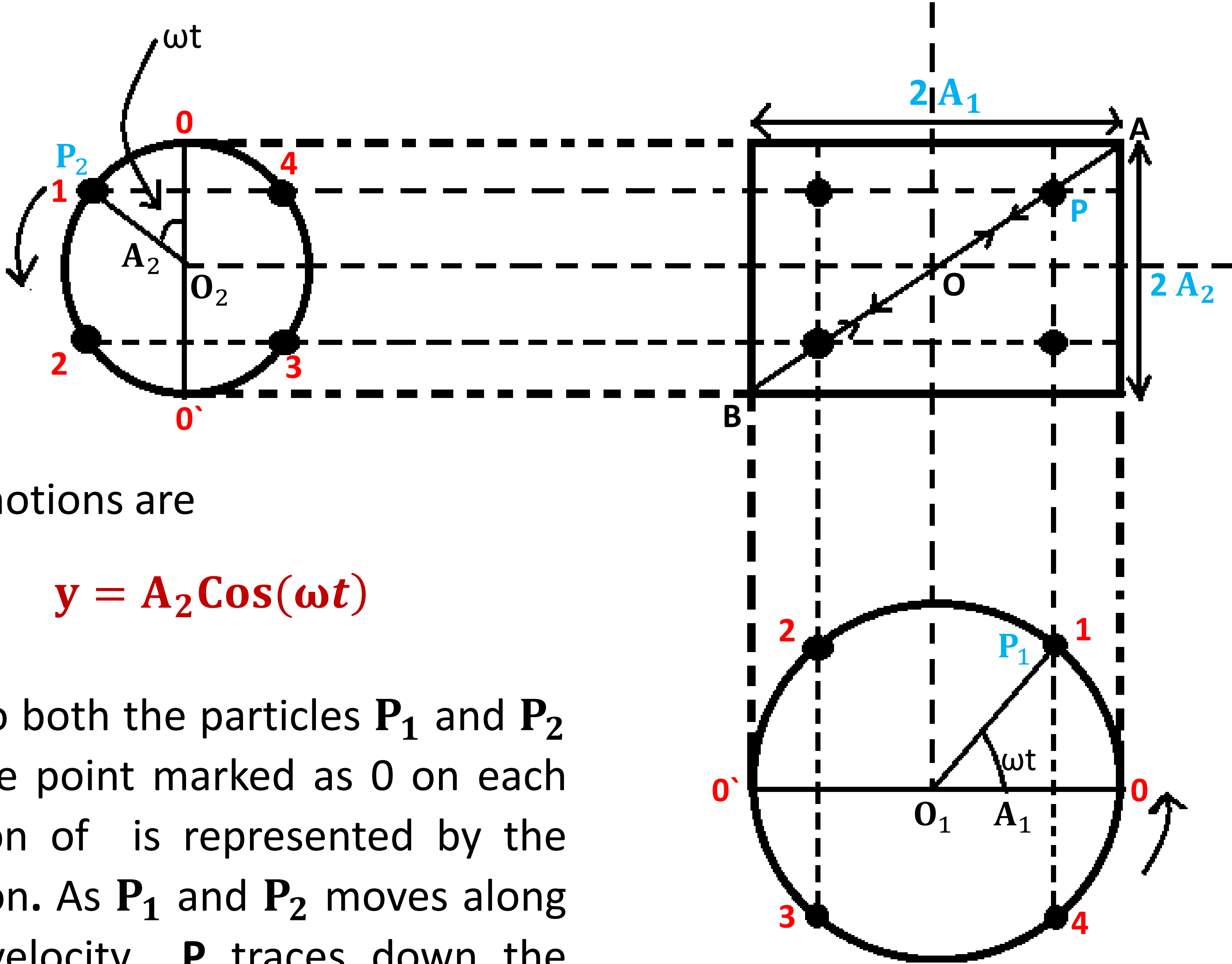
$$\mathbf{x = A_1 Cos(\omega t)} \qquad \& \qquad \mathbf{y = A_2 Cos(\omega t + \phi)}$$

The graphical representation is known as Lissajous figure.

Now we will discuss about some special cases for some specific values of ϕ .

Case-1

$\phi=0$

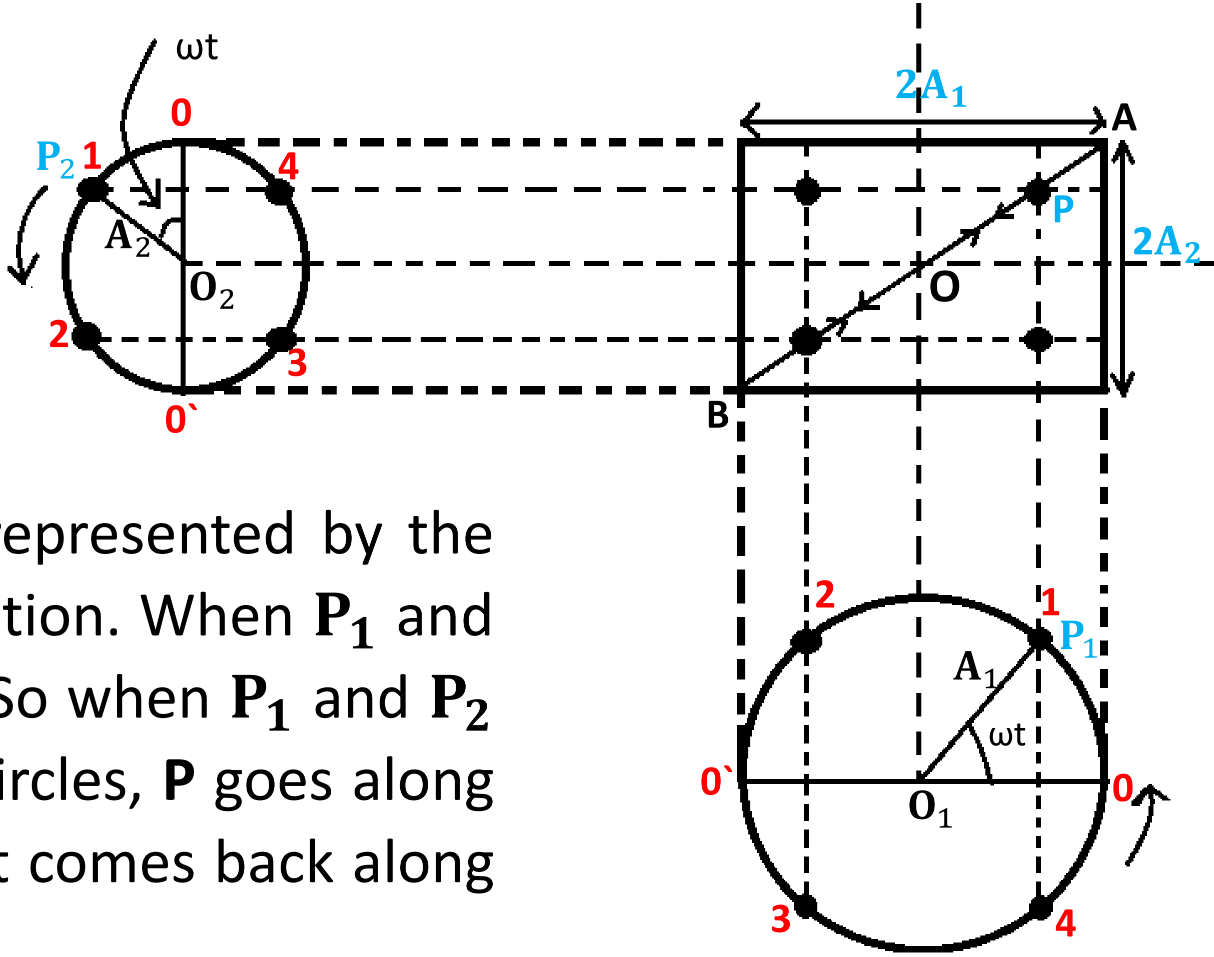


For this case the perpendicular motions are

$$\mathbf{x = A_1 Cos(\omega t)} \qquad \& \qquad \mathbf{y = A_2 Cos(\omega t)}$$

Since the phase difference is zero both the particles $\mathbf{P_1}$ and $\mathbf{P_2}$ starts at the same time from the point marked as 0 on each circle. And the resultant position of is represented by the point **A** on the rectangular section**.** As $\mathbf{P_1}$ and $\mathbf{P_2}$ moves along the circle with same angular velocity **P** traces down the straight

Line passing through the centre of the rectangle. On each circle four positions of P_1 and P_2 have been shown namely 1,2,3,4, which are shown as black dots on each circle.

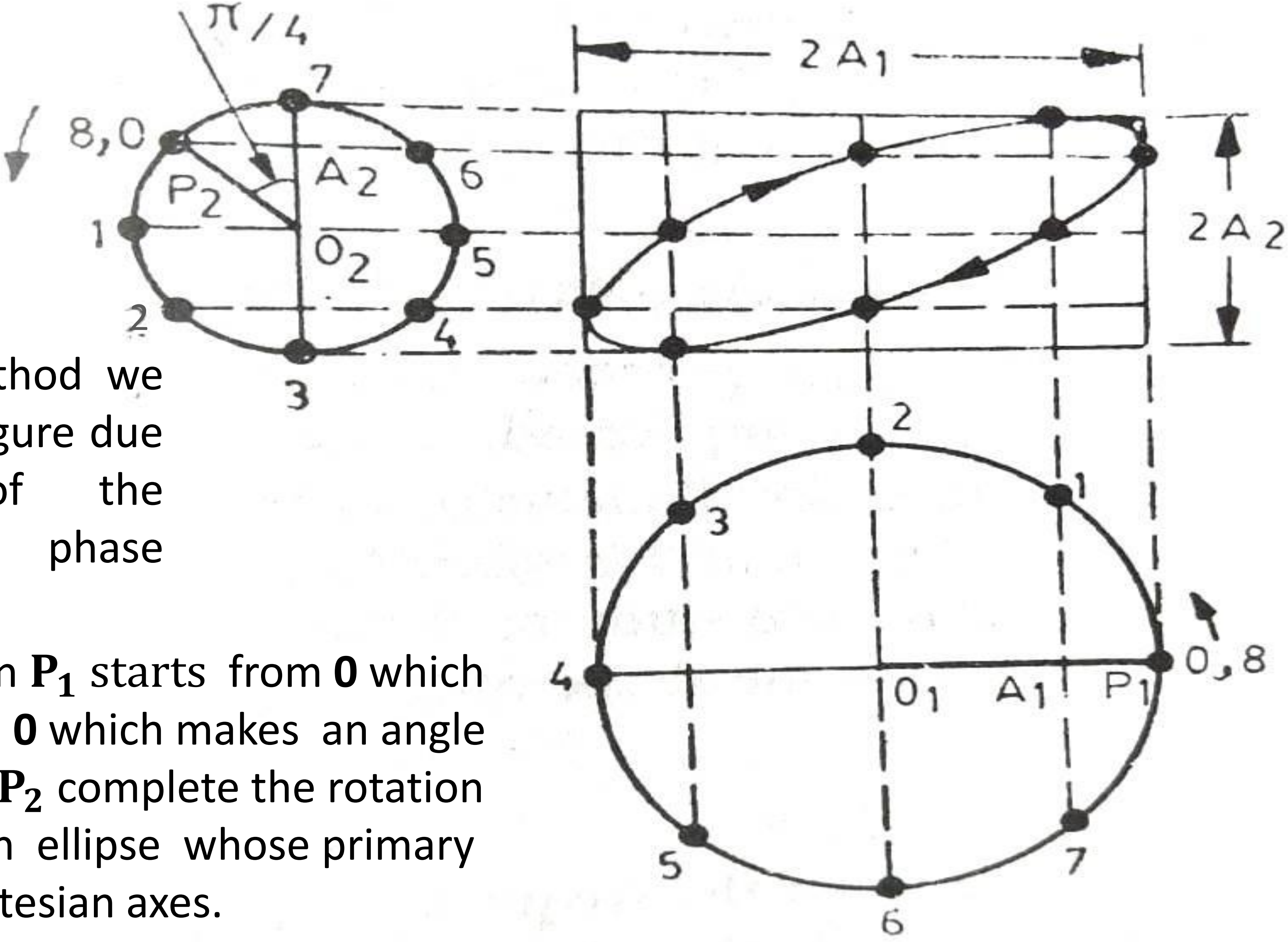


And the corresponding position of **P** has been represented by the black dots(crossing points) on the rectangular section. When P_1 and P_2 reaches the point **0`**, **P** comes to the point **B**. So when P_1 and P_2 complete their rotation around their respective circles, **P** goes along the straight starting from **A** to **B** and then again it comes back along the same path to **A** as shown in the fig.

So the resultant oscillation of two perpendicular SHM with same frequency and zero phase difference will be a pair of straight lines as we have already seen in equation (4) in the previous lecture.

Case-2

ϕ=π/4



Similar to the previous method we can determine the Lissajous figure due to the superposition of the Perpendicular waves having phase difference of π/4.

In this case it is clear that when $\mathbf{P_1}$ starts from **0** which is on the x- axis, $\mathbf{P_2}$ starts from **0** which makes an angle π/4 with the y-axis. As $\mathbf{P_1}$ and $\mathbf{P_2}$ complete the rotation the resultant path becomes an ellipse whose primary Axes are not parallel to the Cartesian axes.

For ϕ=π/2 the principle axes of the ellipse will be parallel to the Cartesian axes.

Superposition of Two Perpendicular Harmonic Oscillations With unequal frequencies

Till now we have seen the superposition of waves having equal frequencies.

In this section we will discuss about the superposition of two perpendicular waves having unequal frequencies. First we will find the shape of Lissajous fig. by analytical calculation and then we will plot the Lissajous figure by rotating vector method graphically. Let us take perpendicular waves having frequency ration 1:2. The equation of the waves are

$$\mathbf{x = A_1 Cos(\omega t)} \qquad \& \qquad \mathbf{y = A_2 Cos(2\omega t + \phi)} \qquad .... 1$$

In analytical method, we find the locus of the instantaneous particle position by eliminating time t from the above equations. Expanding the 2nd equation we get-

$$\frac{y}{A_2} = \mathrm{Cos}(2\omega t)\, cos(\phi) - \mathrm{Sin}(2\omega t)\mathrm{Sin}(\phi)$$

$$\Rightarrow \frac{y}{A_2} = (2\mathrm{Cos}^2\omega t - 1)\, \cos(\phi) - 2\mathrm{Sin}(\omega t)\mathrm{Cos}(\omega t)\mathrm{Sin}(\phi)) \qquad .... 2$$

Now $\quad \mathrm{Cos}(\omega t) = \frac{x}{A_1}$ & $\mathrm{Sin}(\omega t) = \sqrt{1 - \frac{x^2}{A_1^2}}$

Therefore from equation (2) we can write

$$\frac{y}{A_2} = \left(2\frac{x^2}{A_1^2} - 1\right)\cos(\phi) - 2\frac{x}{A_1}\sqrt{1 - \frac{x^2}{A_1^2}}\,\mathrm{Sin}(\phi)$$

$$\Rightarrow \left(\frac{y}{A_2} + \cos\phi\right) - 2\frac{x^2}{A_1^2}\mathrm{Cos}(\phi) = -2\frac{x}{A_1}\sqrt{1 - \frac{x^2}{A_1^2}}\,\mathrm{Sin}(\phi)$$

$$\Rightarrow \boxed{\left(\frac{y}{A_2} + \cos\phi\right)^2 + \frac{4x^2}{A_1^2}\left(\frac{x^2}{A_1^2} - 1 - \frac{y}{A_2}\mathrm{Cos}(\phi)\right) = 0} \quad ....3$$

This is an equation of 4^{th} degree which, in general, represents a closed curve having two loops.

Now we will consider some special cases corresponding to some specific values of ϕ

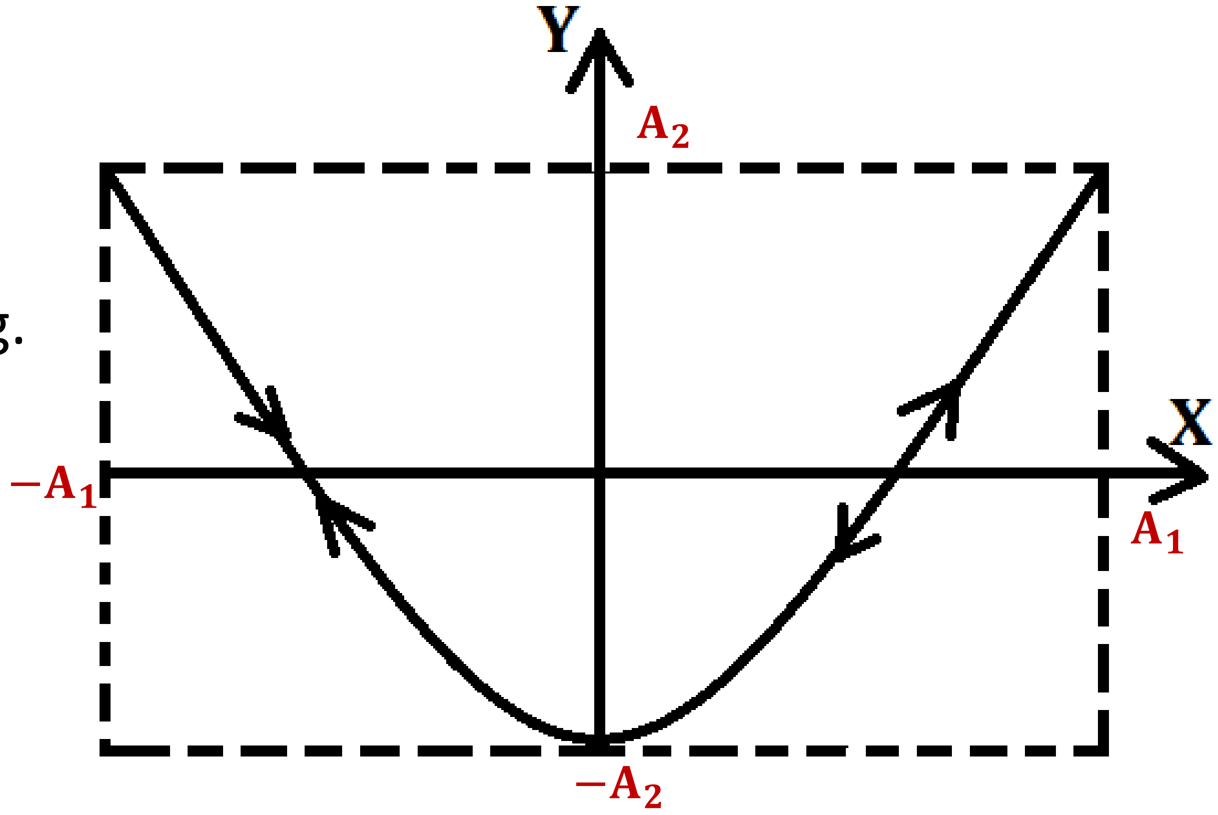
Case-1

$\phi = 0$

For $\phi = 0$ the equation(3) becomes

$$\left(\frac{y}{A_2}+1\right)^2+\frac{4x^2}{A_1^2}\left(\frac{x^2}{A_1^2}-1-\frac{y}{A_2}\right)=0$$

$$\Rightarrow \left(\frac{y}{A_2}+1-\frac{2x^2}{A_1^2}\right)^2=0$$

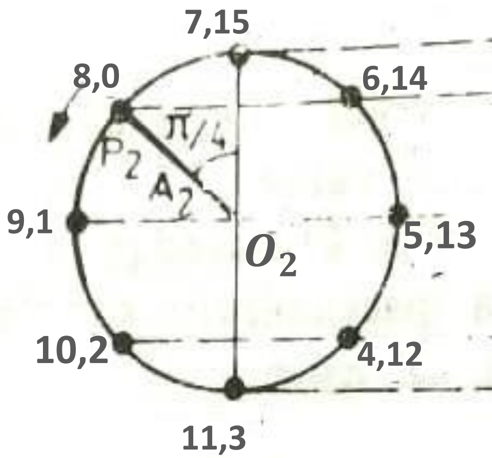


The above equation represents a pair of parabolas With their vertices at $(0, -A_2)$ as shown in the fig. aside.

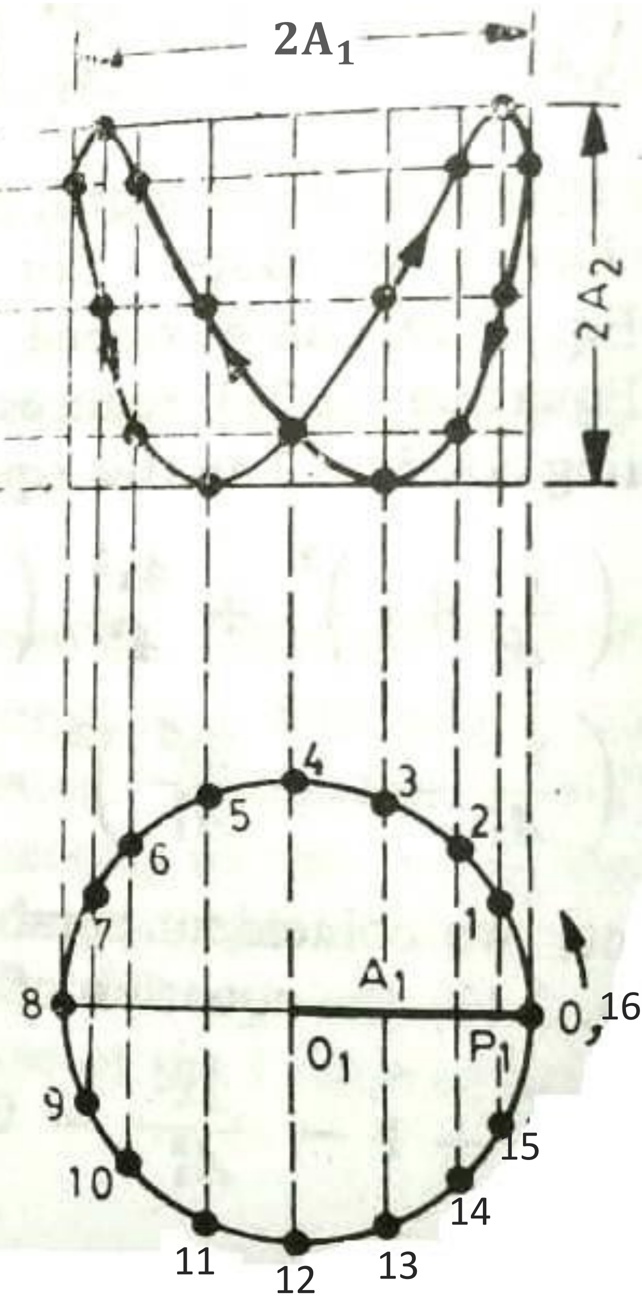
The equation of each parabola becomes-

$$\frac{y}{A_2}+1-\frac{2x^2}{A_1^2}=0 \Rightarrow \boxed{x^2=\frac{A_1^2}{2A_2}(y+A_2)}$$

For other values of ϕ equation (3) becomes very Cumbersome. In those cases we can plot the Lissajous fig. by graphical method quite conveniently.



The fig. aside shows the Lissajous fig using rotating vector method for **$\phi=\pi/4$** and $\omega_2 = 2\omega_1$



The rotating vector **O_2P_2** makes an angle **$\pi/4$** at t=0 with the y-axis but the rotating vector **O_1P_1** is along the x-axis at that instant. The y oscillation is twice as fast as x oscillation and that's why we divide the circle of radius **A_2** into 8 equal parts and circle of radius **A_1** into 16 equal parts. Thus during one complete cycle of ω_2 one goes through only half cycle of ω_1 . And the resultant motion will consist of two close loops as shown in the fig.

For the frequencies with ratio 1:n there will be n number of loops in the Lissajous fig.

Few cases for oscillation with different frequency ratios the Lissajous fig is shown below for $\phi=0$